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BER and Error-Floor Calculation for Multi-Access PCTH

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Abstract—Pseudo-chaotic time hopping (PCTH) is a recently proposed modulation scheme for UWB impulse radio. PCTH exploits concepts from symbolic dynamics to generate aperiodic spreading sequences. In this paper, we present a general analytical expression for the average BER (bit-error-rate) of a synchronous multi-access PCTH system, as a function of the cross-correlation between the users' signatures in an AWGN (additive white Gaussian noise) channel. Also, it is shown that with enough users an error floor in the BER can develop.

I. INTRODUCTION

Over the last decade, there has been a great interest in communication based on impulse radio. These systems make use of ultra-short duration pulses which yield ultra-wideband (UWB) signals characterized by low power spectral densities [1], [2]. UWB systems are particularly promising for short-range wireless communications as they potentially combine reduced complexity with low power consumption, low probability-of-intercept (LPI) and immunity to multipath fading. Namely, in [3], [4], it was first demonstrated that UWB signals do not suffer fading and, therefore, only a small fading margin is required to guarantee reliable communications. The successful deployment of UWB technology depends strongly on the development of efficient multiple access techniques. Existing UWB communication systems employ pseudo-random noise (PN) time hopping for multi-access purposes combined with pulse-position modulation (PPM) for encoding the digital information. An analysis of the multi-user capabilities of such systems has been presented by Scholtz *et al.* in [5], [6].

Recently, it has been suggested that aperiodic codes may be used to reduce the spectral features of the transmitted signal [7]. In this work, we consider the pseudo-chaotic time hopping (PCTH) scheme described in [8]. PCTH exploits concepts from symbolic dynamics [9] to generate aperiodic spreading sequences that, in contrast to fixed (periodic) PN sequences, depend on the input data. The PCTH scheme combines pseudo-chaotic encoding with a multilevel pulse-position modulation. The pseudo-chaotic encoder operates on the input data in a way that resembles a convolutional encoder [10]. Its output is then used to generate the time hopping sequence, resulting in a random distribution of the inter-pulse intervals.

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In this paper we present a multiple access technique for the PCTH communication scheme, that we call MA-PCTH. In MA-PCTH, each pulse transmitted by the original PCTH scheme is replaced by a pulse-train, different for each user. Each pulse-train represents the user "signature", very much like in CDMA (code-division multiple access) schemes [11], but now in the time domain. The signal for each user is demodulated using a pulse correlator followed by maximum-likelihood detection [10].

This paper is organized as follows. In Sec. II, we recall the basic MA-PCTH modulation scheme. In Sec. III, the general expression of the BER performance is presented, followed by the conditions for an error floor in Sec. IV. Simulation results are shown in Sec. V.

II. MULTI-ACCESS PSEUDO-CHAOTIC TIME HOPPING

In this section we recall the basics of PCTH [8] and MA-PCTH [14]. PCTH exploits symbolic dynamics to embed user input data into a pseudo-chaotic sequence. A simple example of a chaotic map is the *Bernoulli shift* [12], defined as:

$$x_{k+1} = 2x_k \mod 1 \qquad (1)$$

The state x can be expressed as a binary expansion:

$$x = 0.b_1 b_2 b_3 \ldots = \sum_{j=1}^{\infty} 2^{-j} b_j, \qquad (2)$$

with b_j equal to either "0" or "1", and $x \in I = [0, 1)$.

In PCTH, the Bernoulli shift (1) is approximated by means of a finite-length (M-bit) shift register, R. The output of the pseudo-chaotic encoder is used to drive a pulse-position modulator (PPM). Each pulse is positioned, dependent on the pseudo-chaotic modulation, within a periodic frame of period T_F. In other words, only one pulse is transmitted within each frame time. If the pulse occurs in the first half of the frame a "0" is being transmitted, otherwise a "1". Each pulse can occur at any of $N = 2^M$ discrete time instants, where M is the number of bits in the shift register, R. The PCTH receiver comprises a pulse correlator, matched to the pulse shape, followed by a pulse-position demodulator (PPD) and a detector. In the simplest case the binary message may be retrieved by means of a threshold detector at the output of the PPD. For more details, see reference [8].
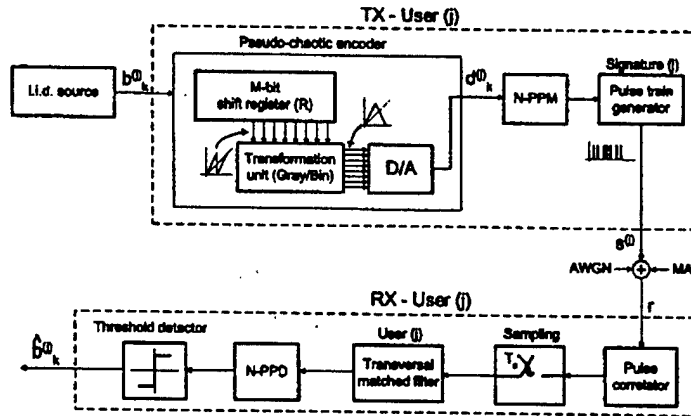
Fig. 1. Simplified block diagram of the MA-PCTH scheme.

Fig. 1 shows a simplified block diagram (for the generic j-th user) of the MA-PCTH multiple access scheme. The input to the system is an i.i.d. (independent identically distributed) source of binary data, $b_k^{(j)}$, where the subscript denotes the k-th bit. The input sequence feeds the pseudo-chaotic encoder whose output, $d_k^{(j)}$, drives the N-PPM modulator producing the time hopping. In MA-PCTH, the output of the modulator is used to trigger a pulse-train generator corresponding to the specific signature, $c^{(j)}$, associated with the j-th user. In this work, we consider a slotted system where the periodic frames (of period T_F) corresponding to the different users are synchronized with each other. Each frame is sub-divided into N slots of duration $T_s(=T_F/N)$. In turn, each slot contains N_c chips; correspondingly, the chip time is given by $T_c = T_s/N_c$. In our analysis we assume that, for each user, the pulse-trains are confined within the slot time T_s, i.e. the user signatures do not invade adjacent slots. This also implies that, within a given frame time T_F, two generic users (j) and (k) will either transmit in different slots or collide. The situation, for a single frame period, is illustrated in Fig. 2.

The transmitted signal, $s^{(j)}(t)$, for the j-th user can be expressed, for each frame, as:

$$s^{(j)}(t) = \sum_{l=0}^{N_c-1} c_l^{(j)} w_p(t - lT_c - d_k^{(j)}T_s), \quad t \in [0, T_F]$$

where $c_l^{(j)} \in \{0, 1\}$ ($l = 0, \ldots, N_c - 1$) is the binary sequence representing the j-th user's signature. $w_p(t)$ is the pulse waveform that in this work is assumed to be rectangular:

$$w_p(t) = \begin{cases} 1, & 0 < t < t_p \\ 0, & otherwise \end{cases}$$

where t_p is the pulse duration, and $t_p < T_c$. So, for each information bit, $b_k^{(j)}$, a pseudo-chaotic iterate $d_k^{(j)} \in \{0, \ldots N - 1\}$ is generated and the pulse-train for the j-th user is transmitted in the corresponding slot, within the frame.

In general, with N_u users transmitting simultaneously, the input to the j-th receiver will be: $r(t) = s^{(j)}(t) + n_u^{(j)}(t) + n(t)$, where the term $n_u^{(j)}(t)$ accounts for the multi-access interference (MAI), caused by the remaining $(N_u - 1)$ users.

Referring to Fig. 1, the j-th receiver comprises a pulse correlator for the pulse waveform $w_p(t)$. The output of the correlator is given for slot s by:

$$\rho_{si} = \int_{iT_c + sT_s}^{(i+1)T_c + sT_s} w_p(\tau) r(\tau) d\tau, \quad i = 0, \ldots, N_c - 1$$

which is sampled at each chip time, T_c. The samples ρ_{si} are then fed into a (digital) transversal matched filter [13]. In the case under consideration, the weights, a_i, should coincide with the user signature, that is: $a_i \equiv c_i^{(j)}$, $i = 0, \ldots, N_c - 1$. Thus, the output of the transversal filter for slot s is:

$$y_s^{(j)} = \sum_{i=0}^{N_c-1} c_i^{(j)} \rho_{si}, \quad s = 0, \ldots, N - 1 \quad (3)$$

where the subscript s runs over the number of slots per frame. The pulse-position demodulation is carried out by applying a maximum-likelihood criterion on each frame. Namely, the
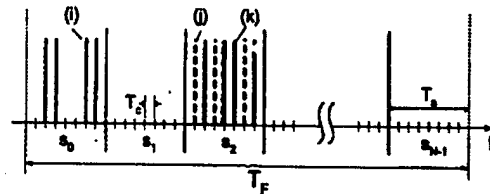

Fig. 2. Sketch of the periodic frame for the MA-PCTH scheme with three users. The frame period, T_F, is divided into N slots, each of duration $T_s(= T_F/N)$. Note the different "signatures" associated with the different users. Users (j) and (k) exhibit a collision in the third slot (s_2).

most likely slot, $\hat{s}^{(j)}$, is:

$$\hat{s}^{(j)} = \arg\max_{s}\{y_s^{(j)}, s = 0, \ldots, N-1\}$$

Finally, the estimate $\hat{b}_k^{(j)}$ of the transmitted bit (for the j-th user) can be obtained by means of a threshold detector.

III. THEORETICAL BIT ERROR RATE

In this section we analyze the bit-error-rate performance of the MA-PCTH scheme. The SNR (signal-to-noise ratio) is defined by E_b/N_0, where E_b is the energy per user bit and N_0 is the single-sided noise power spectral density ($\sigma_n^2 = N_0/2$) of the AWGN (additive white Gaussian noise). We assume that the receiver tries to demodulate the data transmitted by user 1 in the presence of multiple access interference introduced by the $(N_u - 1)$ other users. The cross-correlation value with each user is normalized to the auto-correlation value of user 1. We present a detailed analysis for the three-user case and provide a general BER expression for an arbitrary number of users.

A. Three-User Case

If three users are present, all three can transmit in different slots (event denoted by A), all three can transmit in the same slot (event B), or each of the three possible pairs of users can transmit in the same slot (events C_{12}, C_{23}, C_{13}). Specifically, C_{ij} corresponds to users i and j transmitting in the same slot and the remaining user in a different slot. The average error probability, P_e, of detecting user 1 in the wrong slot is:

$$
\begin{aligned}
P_e =\ & P(error|A)P(A) + P(error|B)P(B) + \\
& + P(error|C_{12})P(C_{12}) + \\
& + P(error|C_{23})P(C_{23}) + P(error|C_{13})P(C_{13})
\end{aligned}
$$

Assuming all users transmit i.i.d. (independent identically distributed) binary data, $P(A) = (N-1)(N-2)/N^2, P(B) = 1/N^2$, and $P(C_{12}) = P(C_{23}) = P(C_{13}) = (N-1)/N^2$. $P(error|A)$ is obtained by modifying the symbol error probability of N-ary orthogonal signaling [10], by considering that users 1, 2 and 3, each transmit in different slots:

$$P(error|A) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[1 - \Phi\left(y - \sqrt{2S_1}\gamma_2\right) \cdot \right.$$
$$\left. \cdot \Phi\left(y - \sqrt{2S_1}\gamma_3\right) \Phi(y)^{N-3}\right] e^{-\frac{(y-\sqrt{2S_1})^2}{2}} dy \quad (4)$$

where: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$, $S_1 = E_1/N_0$ is the SNR of user 1, with E_1 the transmitted energy of user 1, and $\gamma_j = \sum_{i=0}^{N_c-1} c_i^{(1)} c_i^{(j)}$ denotes the periodic cross-correlation between user 1 and j. On the other hand, $P(error|B)$ is obtained by considering that the interference due to user 2 and user 3 appears in the same slot occupied by user 1:

$$P(error|B) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[1 - \Phi(y)^{N-1}\right] \cdot$$
$$\cdot e^{-\frac{(y-\sqrt{2S_1}(1+\gamma_2+\gamma_3))^2}{2}} dy \quad (5)$$

The effect of users 2 and 3 transmitting in the same slot as user 1 can be readily seen from Eq. (5) as effectively improving the SNR and decreasing the error probability of user 1.

The probability of error for the remaining events C_{12}, C_{23}, and C_{13} can be calculated using:

$$P(error|C_{ij}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[1 - \Phi\left(y - \sqrt{2S_1}\gamma_d\right) \cdot \right.$$
$$\left. \cdot \Phi(y)^{N-2}\right] e^{-\frac{(y-\sqrt{2S_1}(1+\gamma_s))^2}{2}} dy \quad (6)$$

where γ_d denotes the total cross-correlation of users transmitting in the same slot, but different from the slot occupied by user 1, and γ_s indicates the total cross-correlation of users occupying the same slot as user 1. Then, $P(error|C_{12})$ is found by setting $\{\gamma_d = \gamma_3, \gamma_s = \gamma_2\}$, $P(error|C_{13})$ is found by setting $\{\gamma_d = \gamma_2, \gamma_s = \gamma_3\}$, while $P(error|C_{23})$ can be obtained by setting $\{\gamma_d = \gamma_2 + \gamma_3, \gamma_s = 0\}$.

B. General case

For N_u users, we generalize the previous considerations to the following interference event denoted by C. There are n slots indexed by $i = 1, \ldots, n$, different from the slot used by user 1, and slot i contains α_i interfering users. The slot occupied by user 1 receives contributions from $N_u - 1 - \sum_{i=1}^{n} \alpha_i$ interferers and all the others slots are not used. The probability that user 1 is detected in the wrong slot is given by [14]:

$$P(error|C) =$$
$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[1 - \prod_{i=1}^{n} \Phi\left(y - \sqrt{2S_1} \sum_{k=1}^{\alpha_i} \gamma^{(i,k)}\right) \Phi(y)^{N-1-n}\right] \cdot$$
$$\cdot \exp\left[-\frac{1}{2}\left(y - \sqrt{2S_1}\left(1 + \sum_{k=1}^{N_u-1-\sum_{i=1}^{n}\alpha_i} \gamma^{(k)}\right)\right)^2\right] dy$$
$$(7)$$

where $\gamma^{(i,k)}$ represents the cross-correlation between user 1 and the interferer indexed by k, in the slot indexed by i, while $\gamma^{(k)}$ is the cross-correlation between the interfering user indexed by k, and user 1.

In order to calculate the average probability of error in the general case, we need an expression for the probability of each of the possible interference events. The average probability of error is:

$$P_e = P(error|A)P(A) + P(error|B)P(B) + P_e(C') \quad (8)$$

A denotes the event where all users transmit in the same slot, B is the event where all users transmit in different slots, and C' denotes the collection of all other interference events. It follows that: $P(A) = (N-1)(N-2)\cdots(N-(N_u-1))/N^{N_u-1}$,

and $P(B) = 1/N^{N_u-1}$. Moreover,

$$P(error|A) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 1\left[-\Phi(y)^{N-N_u} \cdot \right. \tag{9}$$
$$\left. \cdot \prod_{j=2}^{N_u} \Phi\left(y - \sqrt{2S_1}\gamma_j\right) \right] e^{-\frac{(y-\sqrt{2S_1})^2}{2}} dy$$

$$P(error|B) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} 1\left[-\Phi(y)^{N-1} \right] \cdot \tag{10}$$
$$\cdot e^{-\frac{\left(y-\sqrt{2S_1}\left(1+\sum_{j=2}^{N_u}\gamma_j\right)\right)^2}{2}} dy$$

where again γ_j denotes the periodic cross-correlation between users 1 and j. In practice, for equal cross-correlations $\gamma_j = \gamma$, $P_e(C')$ can be calculated as the weighted average of the probability of error of all interference events (except A and B):

$$P_e(C') = \frac{1}{N^{N_u-1}} \sum_{\lambda=1}^{\lambda_{max}} \sum_{a_\lambda=2}^{a_{\lambda-1}} \cdots \sum_{a_3=2}^{a_2} \sum_{a_2=2}^{a_1} \sum_{a_1=2}^{a_0}$$
$$\binom{N_u}{a_1}\binom{N_u-a_1}{a_2}\cdots\binom{N_u-a_1-\ldots-a_{\lambda-1}}{a_\lambda} \cdot$$
$$(N-1)(N-2)\ldots(N-(N_u-\sum_{i=1}^{\lambda-1}a_i)-(\lambda-1)) \cdot$$
$$\left(\frac{1}{\beta_2!\beta_3!\ldots}\right)\frac{1}{N_u}\left[a_1 P_e(A_1^1, A_2, \ldots, \alpha_0) + \right.$$
$$\left. a_2 P_e(A_1, A_2^1, \ldots, \beta_0) + \ldots + \alpha_0 P_e(A_1, A_2, \ldots, \alpha_0^1)\right] \tag{11}$$

where, $a_0 = min(N_u - 1, N_u - \sum_{i=2}^{\lambda} a_i)$. β_2 is the number of slots in which two users transmitted, and β_3 is the number of slots in which three users transmitted, etc. In Eq. (11), $\lambda_{max} = \lfloor(\frac{N_u}{2})\rfloor$ is the maximum number of different possible interference events within a single frame. This occurs when $N_u/2$ pairs of users interfere. The number of users who transmit in slots with no interfering users is $\alpha_0 = N_u - \sum_{i=1}^{\lambda} a_i$. In the above expression, the events $A_1, \ldots, A_\lambda$ correspond to a_1 users transmitting in the same slot, a_2 users transmitting in the same slot but different than the A_1 users, etc. The superscript 1 indicates that user 1, the user of interest is included in that set.

For all cases, in order to calculate the BER, we need to convert symbol errors to bit errors. Namely, we convert the probability of detecting user 1 in a wrong slot, P_e, to the bit error probability P_b. The errors which consist of confusing the slot used by user 1 with any of the other $N - 1$ slots are equiprobable and occur with probability: $P_e/(N - 1) = P_e/(2^M - 1)$. Let's assume, without loss of generality, that the binary information digit transmitted by user 1 is zero; then the probability that the receiver makes a bit error is the probability of confusing the slot where user 1 is transmitting with any of the last $N/2$ slots in the frame. Thus,

$$P_b = \frac{2^{M-1}}{2^M-1} P_e \approx \frac{P_e}{2}, \quad M \gg 1 \tag{12}$$

IV. Error Floor Condition

If only two users are present, for each frame, they transmit either in the same slot or in different slots. The maximum interference, due to the total cross-correlation, γ_T, has just a single term, that due to user 2. If, as required to discriminate among different users, the cross-correlation between users 1 and 2 is less than unity no error floor is present. For more than 2 users, the condition for the existence of an error floor is:

$$\gamma_T = \sum_{j=2}^{N_u} \gamma_j \geq 1 \tag{13}$$

In this work we chose $\gamma_j = \gamma = 0.5625$, so three users is the minimum number of users for which an error floor develops, since $\gamma_T = 1.125$. The event causing the error floor is C_{23} in Sec. III-A. The error floor occurs because $P(error|C_{23}) \to 1$ as SNR $\to \infty$. This can be seen from Eq. (6) which for C_{23} has $\gamma_d = \gamma_T = 1.125$, and $\gamma_s = 0$. The error floor value is then given by: $P(C_{23}) = (N-1)/N^2$.

If three or more users are present, the value of the error floor is the sum of the event probabilities, $P(C')$, where the condition (13) is met. It follows that the error floor can be expressed as:

$$P_{e,floor} = P_e(C')\Big|_{SNR\to\infty} \tag{14}$$

V. Simulation Results

This section reports the BER simulation results for the MA-PCTH scheme and compares them with the theory.

In our previous work we have shown that the BER performance improves with decreasing cross-correlation [14]. This is consistent with the fact that orthogonal signaling results in the best possible BER performance.

In the simulations we used $M = 8$ bits corresponding to $N = 256$ PPM levels, with $N_c = 32$ chips/slot. In each of the multi-user cases, a 32-bit signature sequence was assigned to the different users. The binary sequences that we chose to use were randomly selected. One constraint imposed on the sequence selection process was that each sequence should contain an equal number of ones (specifically 16 ones and 16 zeros). This maintains a constant energy across all users. The randomly selected sequences have a periodic cross-correlation value to user 1, the user of interest, of 0.5625. Fig. 3 shows the analytically calculated (using Eq. (8)) and simulated two-, three-, and four-user bit-error-rate curves. Since the maximum cross-correlation in the two-user case does not meet the criterion of Eq. (13), no error floor is present (see Fig. 3(a)). With γ set to 0.5625, Eq. (13) is first satisfied for three users. The maximum cross-correlation, γ_T, in the three-user case is $2\gamma = 1.125$. The three-user case is shown in Fig. 3(b) where the error floor is due to the event C_{23}, as discussed in Sec. III-A and Sec. IV. The value of the error floor is the corresponding coefficient, $P(C_{23}) = (N-1)/N^2$. Converting from symbol error
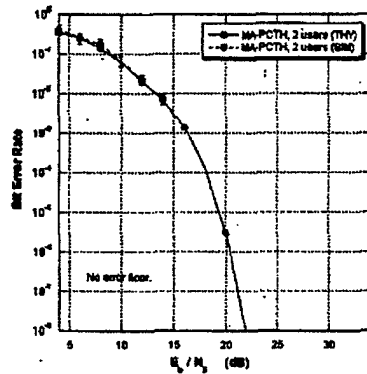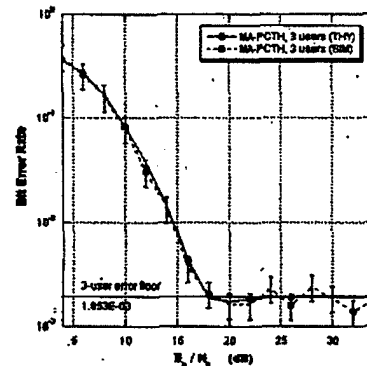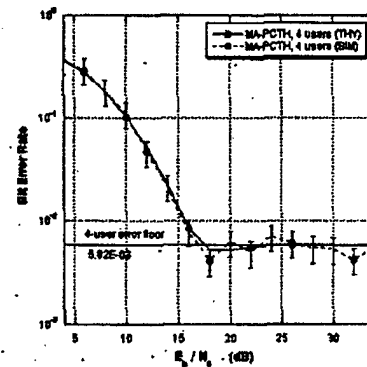
rate to BER using, $P_b(C_{23}) = \frac{2^{M-1}}{2^M-1}P_e(C_{23})$, we find the floor to be 1.953E-03. Eq. (11) can be used to calculate the expected performance of the 4-user case. Fig. 3(c) shows the simulated and calculated performance with $\gamma_2 = \gamma_3 = \gamma_4 = 0.5625$. Notice that the error-floor value increases with the number of users.

VI. CONCLUSIONS

The success of MA-PCTH as a communication system depends on how many users can be supported at a sufficiently low error rate and a sufficiently high data rate. In this paper we have presented a general expression for the average probability of error for synchronous MA-PCTH. We have shown that the BER is dependent on the total cross-correlation and thus the number of users. Also, an error floor exists if the total cross-correlation exceeds one.

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(a)

(b)

(c)

Fig. 3. Simulated and analytical BER performance of the two-, three-, and 4-user cases of the MA-PCTH scheme ($\gamma = 0.5625$).